To Maria

# INTRODUCTION

The development of computers makes it possible to create numerical models of various objects and to perform experiments with them. Numerical models are used in systems that are used for design (CAD), engineering (CAE), and manufacturing (CAM) of the modeled object. In all these systems it is necessary to describe the geometric shape of the modeled object up to certain precision. During the development of these systems a new branch of mathematics called “geometric modeling” was created. Geometric modeling studies the methods used to construct numerical geometric models of real and imaginary objects, as well as methods to control these models.

A geometric model consists of a shape description for the modeled object along with a description of the relationships among the model elements. To allow changes of these models and creation of similar models, the construction tree describing the sequence and methods used in model construction is frequently included with the model. The elements of a geometric model are usually provided by attributes carrying information about their physical and other properties.

The development of geometric modeling began with computer drawing systems. Wireframe and surface modeling came later on. Parametric and solid modeling systems have dramatically changed the way designers work. They made it possible for designers to capture their ideas in three-dimensional models and not just in flat drawings. To describe the relationship of model elements, variational methods have been used. Another step in the development of geometric modeling was direct modeling, which further empowered engineers and designers by circumventing the constraints of history-based modelers.

The objects around us occupy a finite amount of space. In order to model these objects we need to describe the part of the space they occupy. In certain cases it can be done using the volume elements of the modeled object. These volume elements are called voxels. Often such volume elements are cubes, prisms, or pyramids. These models are used in the cases where the attributes of the volume elements are more important than the geometric shape of the whole model.

Within certain accuracy limitations, the geometric shape of an object can be described using flat faces. This kind of description is called polygonal representation. A polygonal representation describes curved surfaces using a set of triangular or quadrangular plates. Usage of flat faces significantly simplifies work with the model. Polygonal models are typically based on measurements of real objects, or on other models. Polygonal representation is widely used for visualization of geometric models.

Many objects can be constructed using translational and rotational motion. Surface elements of such objects can be specified by a plane, a spherical surface, a

cylindrical surface, a conical surface, or a toroidal surface. All of these surfaces divide the space into two parts, and one can specify on which side of the surface the internal volume of the modeled object is situated. With the help of these surfaces, it is possible to construct a geometric model by performing operations on primitives. The primitives usually include a rectangular prism, a triangular prism, a sphere, a cylinder, a cone, and a torus. This approach is called constructive solid geometry. Nowadays constructive solid geometry is rarely used.

The most general way to describe the geometric form of a modeled object is to use free-form surfaces, represented explicitly. In this approach, a surface representation of the modeled object is given by a set of faces joined along the edges. The faces contain information about their boundaries and relations to their neighbors. They are joined so that the outer side of one face merges into the outer side of the adjacent face. Faces may have an arbitrary form. This approach to describing a modeled object is called boundary representation. It makes it possible to perform many operations on models while maintaining a standard representation of their internal organization. A boundary representation contains a precise description of the boundary of the modeled object that separates it from the rest of the space. Boundary representation is used in all modern CAD systems.

Variational relations are established between the elements of a geometric model to align points, axes, and surfaces, as well as to specify tangency and many other relations. Interdependencies of model elements are called geometric constraints. Different dimensions of one or several objects may be related to each other. Using these relations one can easily edit the model and create similar models. Typically, there is more than one way to satisfy a set of geometric constraints. Most computer-aided design systems have modules that impose geometric constraints on the elements of the geometric model. Geometric modeling studies methods of finding a solution that meets the demands posed by an actual problem.

Geometric modeling relies on different areas of mathematics, primarily differential geometry and numerical methods. Geometric modeling is closely related to programming; it takes advantage of the features of object-orientation: encapsulation, inheritance, polymorphism. For example, object-oriented programming allows creation of curves and surfaces that have some set of common methods, hiding the implementation of these methods for each particular curve and surface as well as data used to create them.

The geometric model is used in rendering the modeled object, checking the correct assemblage of the object from its elements, performing kinematic verification, calculating inertial characteristics, computing the trajectory of cutting tools, designing equipment, and other preliminary stages of modeled object production. Geometric models can help carry out numerical experiments and manufacture the modeled object. Attributes of the model elements describing physical and other properties of the modeled object are used for this purpose.

Geometric modeling can reduce the time and material costs for production of designed objects and improve their quality. Geometric modeling automates the work of designers, engineers, architects, and technologists, enabling them to avoid routine operations and concentrate on the creative aspects of their work.